AP-C Objectives (from College Board Learning Objectives for AP Physics)

- 1. Motion in two dimensions, including projectile motion
  - a. Students should be able to add, subtract, and resolve displacement and velocity vectors, so they can:
    - i. Determine components of a vector along two specified, mutually perpendicular axes.
    - ii. Determine the net displacement of a particle or the location of a particle relative to another.
    - iii. Determine the change in velocity of a particle or the velocity of one particle relative to another.
  - b. Students should understand the general motion of a particle in two dimensions so that, given functions **x**(**t**) and **y**(**t**) which describe this motion, they can determine the components, magnitude, and direction of the particle's velocity and acceleration as functions of time.
  - c. Students should understand the motion of projectiles in a uniform gravitational field, so they can:
    - i. Write down expressions for the horizontal and vertical components of velocity and position as functions of time, and sketch or identify graphs of these components.
    - ii. Use these expressions in analyzing the motion of a projectile that is projected with an arbitrary initial velocity.
  - d. Students should understand frames of reference, so they can:
    - i. Analyze the uniform motion of an object relative to a moving medium such as a flowing stream.
    - ii. Analyze the motion of particles relative to a frame of reference that is accelerating horizontally or vertically at a uniform rate.

Objectives

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2. Students should understand the general motion of a particle in two dimensions so that, given functions **x**(**t**) and **y**(**t**) which describe this motion, they can determine the components, magnitude, and direction of the particle's velocity and acceleration as functions of time.

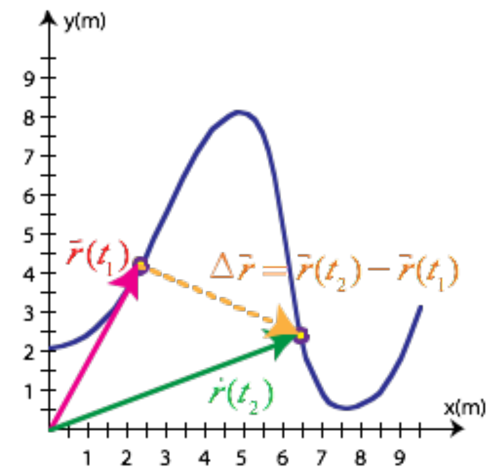
Position Vector

The position vector, **r**, always has its tail at the origin and its head at the current position of the object. The position vector can be written as both **r** and **s**.

$$\vec{r}(t)=x(t)\hat{i}+y(t)\hat{j}=<x(t),y(t)> \qquad \vec{v}_{avg}=\frac{\Delta\vec{r}}{\Delta t}$$

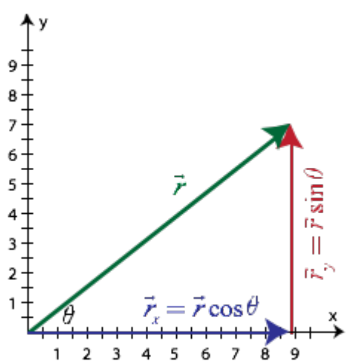
$$\vec{v}=\lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t}=\frac{d\vec{r}}{dt}=\frac{dx}{dt}\hat{i}+\frac{dy}{dt}\hat{j}=<\frac{dx}{dt},\frac{dy}{dt}>$$

$$\vec{a}=\frac{d\vec{v}}{dt}=\frac{d^2x}{dt^2}\hat{i}+\frac{d^2y}{dt^2}\hat{j}=<\frac{d^2x}{dt^2},\frac{d^2y}{dt^2}>$$



Vector Components

Vectors can be expressed as components along mutually perpendicular axes. Manipulating from one form to another is an important skill in analysis of vectors.

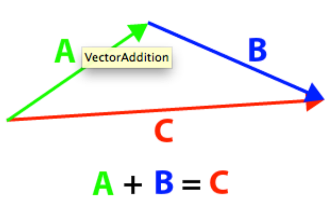


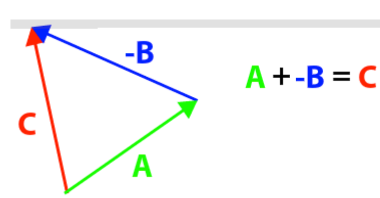
Vector Addition and Subtraction

Graphic vector addition: Line up all vectors tip to tail, and draw a line from the starting point of the first vector to the ending point of the last vector.

Analytic vector addition: Sum up all components separately (x-components, y-components, etc.)

Vector subtraction: Add the opposite of the vector.





Objectives

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Projectile Motion in a Uniform Gravitational Field

Horizontal acceleration is 0 (neglect friction)

Vertical acceleration is g (on Earth's surface, 9.81 m/s^2 down)

Utilize constant acceleration kinematic equations

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

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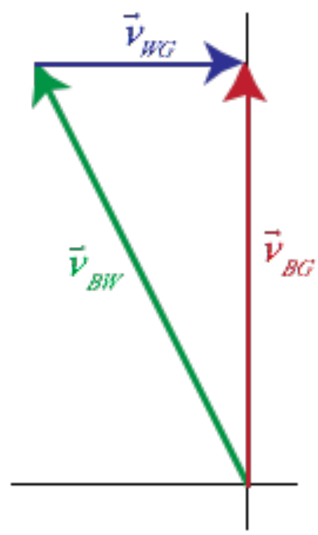
$$v_{AC} = v_{AB} + v_{BC}$$

Example Problem (2D)

A sailor wants to sail his boat due north. The speed of the boat relative to the water is 20 km/h and the water flows from west to east at 9 km/h. In which direction should the boat head? How fast does the boat travel relative to the ground?

$$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$$

velocity of water with respect to ground
velocity of boat with respect to water
velocity of boat with respect to ground



$$\vec{v}_{BW} = 20 \, {}^{km}\!/_{hr}$$

$$\vec{v}_{WG} = 9 \, {}^{km}\!/_{hr}$$

$$\left|\vec{v}_{BG}\right| = \sqrt{(20 \, {}^{km}\!/_{hr})^2 - (9 \, {}^{km}\!/_{hr})^2} = 17.9 \, {}^{km}\!/_{hr}$$

$$\theta = \tan^{-1}\left(\frac{opp}{adj}\right) = \tan^{-1}\left(\frac{9 \, {}^{km}\!/_{hr}}{17.9 \, {}^{km}\!/_{hr}}\right) = 26.7^\circ$$